別紙様式 8

研 究 主 論 文 抄 録

論文題目　Oscillation and nonoscillation criteria for a class of nonlinear ordinary differential equations

ある非線形常微分方程式に対する振動及び非振動条件について

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主論文要旨

In this dissertation we consider the investigation of the oscillation and nonoscillation properties of the functional differential equation with deviating argument of the type

(A) $$(p(t)|x^{(n)}(t)|^{\alpha}\operatorname{sgn} x^{(n)}(t))^{(n)} + q(t)|x(g(t))|^{\beta}\operatorname{sgn} x(g(t)) = 0$$

and their generalizations, where α and β are distinct positive constants and $p(t)$, $q(t)$ are positive continuous functions on $[a, \infty)$, $a > 0$. This dissertation consists three chapters.

Chapter 1. Oscillation and nonoscillation criteria for even order nonlinear functional differential equations;

Chapter 2. Effect of nonlinear perturbation on second order linear nonoscillatory differential equations;

Chapter 3. Asymptotic analysis of positive solutions of second order nonlinear functional differential equations in the framework of regular variation.

We begin with the outline of Chapter 1 which is devoted to the study of equation (A) under the assumption that $p(t)$ satisfies

$$\int_a^{\infty} \frac{dt}{p(t)^{\frac{1}{\alpha}}} = \infty.$$

Our study in this chapter was motivatied by the obserbation that little analysis has been made of functional differential equations involving the differential operator $(p(t)|x^{(n)}(t)|^{\alpha}\operatorname{sgn} x^{(n)}(t))^{(n)}$ from the viewpoint of oscillation and nonoscillation. In Sections 1.1 – 1.3 we classify the set of all possible positive solutions of equation (A) into a finite number of subclass according to their asymptotic behavior as $t \to \infty$:

$$\text{(i)}\ \lim_{t\to\infty} \frac{x(t)}{\varphi_k(t)} = \text{const} > 0,\quad \text{(ii)}\ \lim_{t\to\infty} \frac{x(t)}{\varphi_k(t)} = 0,\ \lim_{t\to\infty} \frac{x(t)}{\varphi_{k-1}(t)} = \infty \ \text{and}\ \text{(iii)}\ \lim_{t\to\infty} \frac{x(t)}{\varphi_{k-1}(t)} = \text{const} > 0,$$

where the functions $\varphi_j(t)$, $j = 0, 1, 2, \cdots, 2n-1$ are defined by

$$\varphi_j(t) = (t-a)^j, \quad j \in \{0, 1, \cdots, n\}$$

and

$$\varphi_j(t) = \int_a^t (t-s)^{n-1} \left[\frac{(s-a)^{j-n}}{p(s)}\right]^{\frac{1}{\alpha}} ds, \quad j \in \{n+1, n+2, \cdots, 2n-1\}.$$

Since the above cases (i) and (iii) are of the same category, we conclude that any positive solution $x(t)$ of equation (A) satisfies either

$$\lim_{t\to\infty} \frac{x(t)}{\varphi_j(t)} = \text{const} > 0 \quad \text{for some}\ j \in \{0, 1, \cdots, 2n-1\}$$

or

$$\lim_{t\to\infty} \frac{x(t)}{\varphi_{k-1}(t)} = \infty \quad \text{and} \quad \lim_{t\to\infty} \frac{x(t)}{\varphi_k(t)} = 0 \quad \text{for some}\ k \in \{1, 3, \cdots, 2n-1\},$$

which means that it is natural to decompose the set P of all positive solutions of (A) into the two classes $P(\mathrm{I})$ and $P(\mathrm{II})$ defined by

$$P(\mathrm{I}) = P(\mathrm{I}_0) \cup P(\mathrm{I}_1) \cup \cdots \cup P(\mathrm{I}_{2n-1}) \quad \text{and} \quad P(\mathrm{II}) = P(\mathrm{II}_1) \cup P(\mathrm{II}_3) \cup \cdots \cup P(\mathrm{II}_{2n-1}).$$

Next, we first derive the integral representation for a solution $x(t) \in P(\mathrm{I}_j)$, $j \in \{0, 1, \cdots, 2n-1\}$, and then derive one for a solution $x(t) \in P(\mathrm{II}_k)$, $k \in \{1, 3, \cdots, 2n-1\}$. Taking into account of the integral equation derived for each subclass of $P(\mathrm{I})$ and $P(\mathrm{II})$, we establish necessary and sufficient conditions for the existence of positive solutions belonging to $P(\mathrm{I})$ on the one hand, and sufficient conditions for the existence of solutions belonging to $P(\mathrm{II})$ on the other hand. In Section 1.4 we derive the comparison principles which relate the oscillation (or nonoscillation) of functional differential equation (A) to that of suitably associated differential equations with or without deviating arguments. In Section 1.5 by using the comparison principles of Section 1.4 effectively, we establish criteria for oscillation of all solutions of equation (A) for the super-half-linear ($\alpha \leqq 1 < \beta$) and sub-half-linear ($\beta < 1 \leqq \alpha$) cases. Extensive use is made of known oscillation results for the companion ordinary differential equation

$$\text{(B)} \qquad (p(t)|x^{(n)}(t)|^{\alpha} \operatorname{sgn} x^{(n)}(t))^{(n)} + q(t)|x(t)|^{\beta} \operatorname{sgn} x(t) = 0.$$

The obtained oscillation criteria are shown to be sharp for restricted classes (classes of regularly varying function) of the function $p(t)$ and the deviating argument $g(t)$.

We will give a brief explanation of the result of Chapter 2. Consider the perturbed second order linear differential equation of the form

$$\text{(C)} \qquad (p(t)x')' + q(t)x + Q(t)|x|^{\gamma} \operatorname{sgn} x = 0,$$

where γ is a positive constant with $\gamma \neq 1$, where γ is a positive constant with $\gamma \neq 1$, and $p(t)$, $q(t)$ and $Q(t)$ are continuous functions on $[a, \infty)$, $a \geqq 0$ such that $p(t) > 0$ and $Q(t) \geqq 0$ for $t \geqq a$. Assume that the second order linear differential equation (D) $(p(t)x')' + q(t)x = 0$ is nonoscillatory, that is, all of its nontrivial solutions are nonoscillatory. It is natural to expect that (C) inherits the nonoscillatory character from (D) as long as the perturbation $Q(t)|x|^{\gamma} \operatorname{sgn} x$ remains "small", and that application of a "sufficiently large"perturbation might turn (D) into an oscillating system (C), which means the oscillation of all of its solutions.

The objective of this Chapter is to verify the truth of this expectation by presenting the results on nonoscillation and oscillation of (D) implying, respectively, preservation of nonoscillation and generation of oscillation of (D), which are determined by the convergence or divergence of the integrals

$$\int_a^{\infty} Q(t)u(t)^{\gamma}v(t)dt \quad \text{and} \quad \int_a^{\infty} Q(t)u(t)v(t)^{\gamma}dt,$$

where $\{u(t), v(t)\}$ is a fundamental system of solutions of (D) consisting of a (uniquely determined) principal solution $u(t)$ and a non-principal solution $v(t)$. In Section 2.2, in order to gain useful information about the structure of nonoscillatory solutions of (C), we classify the set of nonoscillatory solutions of (C) into three types according to their asymptotic behavior at infinity. Next, in Section 2.3 we give sharp conditions for the existence of solutions belonging to these types. Some examples illustrating the nonoscillation theorems obtained are also presented. Finally, in Section 2.4 we consider the case where the function $Q(t)$ in equation (C) is of alternating sign and show that the results of Section 2.3 can be extended to this case. Moreover, we establish sharp oscillation criteria for (C) on the basis of oscillation and nonoscillation results due to the results of Belohorec and Kiguradze.

Finally, the results of Chapter 3 will be sketched. In this Chapter we devote to the study of the existence and asymptotic behavior of positive solutions of second order Emden-Fowler type functional differential equations of the form

$$\text{(E)} \qquad x''(t) + q(t)|x(g(t))|^{\gamma} \operatorname{sgn} x(g(t)) = 0,$$

where γ is a positive constant less than 1, and $q(t)$ is a positive continuous function on $[a, \infty)$ and $g(t)$ is a positive continuous increasing function on $[a, \infty)$ to satisfy $g(t) < t$ and $\lim_{t \to \infty} g(t) = \infty$. It is known that any positive solution $x(t)$ falls into one of the following three types:

(i) $\lim_{t\to\infty} x(t) = \text{const} > 0$, (ii) $\lim_{t\to\infty} x(t) = \infty$, $\lim_{t\to\infty} \frac{x(t)}{t} = 0$ and (iii) $\lim_{t\to\infty} \frac{x(t)}{t} = \text{const} > 0$.
Our primary concern in this chapter will be with type (II)-solutions, which are referred to as "intermediate solutions" of (E), because the other two types of solutions are fully understood. It is shown that if $q(t)$ is regularly varying and $g(t)$ satisfies $\lim_{t\to\infty} g(t)/t = 1$, then one can acquire full knowledge of the structure of all possible regularly varying solutions of relation

$$\text{(F)} \qquad x(t) \sim \int_a^t \int_s^\infty q(r)x(g(r))^\gamma dr ds \quad \text{as } t \to \infty,$$

and that the results for (F) thus obtained play a central role in establishing the existence of intermediate solutions with accurate asymptotic behavior at infinity for equation (E).

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